Mean orbit of the stream of meteoroids

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Mean parameters of the meteoroid stream

Mean orbit of N meteoroids is often calculated as arithmetic mean:

$$<\epsilon>=\frac{1}{N}\sum_{k=1}^{N}\epsilon_k$$

where $\epsilon = a, q, e, \omega, \Omega, i, \pi$.

Mean parameters of the meteoroid stream

Drawbacks:

- Voloshchuk & Kashcheev (1999), Williams (2001)

$$q_m = \frac{1}{N}\sum_{i=1}^{N} q_i \qquad q_p = a_m(1 - e_m)$$

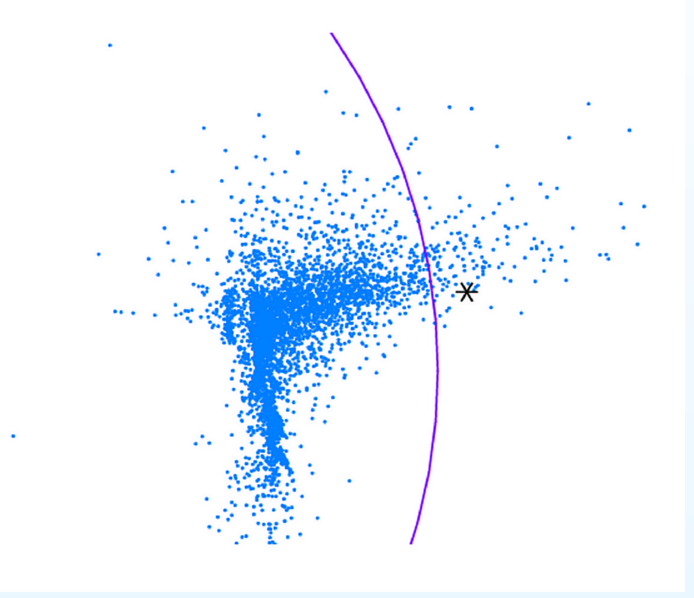
$$\mathbf{q_p} \neq \mathbf{q_m}$$

- Voloshchuk & Kashcheev (1999)

$$0.983 < R_{node} = \frac{q(1+e)}{1 \pm e\cos\omega} < 1.017 \ \ [AU]$$

Mean parameters of the meteoroid stream

- Mean stream is based on sample observed in selection conditions.



  Earth colides with particles of the stream moving through very narrow quasi-cylinder, which is randomly set relatively to toroidal set of stream.

- $R_\Omega \notin (0.983, 1.017)$

Mean parameters of the meteoroid stream

- unknown epoch of the mean parameters,
- there is internal numerical inconsistency between the mean helio- and mean geocentric parameters of the same meteoroid stream.

Mean parameters of the meteoroid stream

Example:

- Let one orbit has: $e_1 = 0.999$, $q_1 = 0.91$
- and another: $e_2 = 1.000991$, $q_2 = 0.85$

Then if:

- $$a_s = \left(\frac{1}{2}\left(\frac{1}{a_1} + \frac{1}{a_2}\right)\right)^{-1} = -29881.1$$
- $$a_s = \frac{a_1 + a_2}{2} = 26.14$$

Proposition (Voloshchuk & Kashcheev, 1999)

Estimation of mean stream orbit based on mean values of parameters taken directly from observations:

- geocentric parameters of radiant α_G, δ_G, V_G and $\lambda_\oplus$
- mean V_G and δ_G calculated as a weighted mean:

$$\epsilon_m = \frac{\sum_k w_k \epsilon}{\sum_k w_k}$$

where

$$w_k = \begin{cases} (1 - \frac{D_{mk}}{D_o})^2, & D_{mk} \leq D_o \\ 0, & D_{mk} > D_o \end{cases}$$

where D_{mk} is the orbital similarity function, and D_o is the similarity threshold.

D_{mk} is estimated according to Southworth & Hawkins formula.

Proposition (Voloshchuk & Kashcheev, 1999)

- the angular parameters α_G and $\lambda_\oplus$ find as solution of the system of equations (Mardia, 1978):

$$C = r\cos\beta$$

$$S = r\sin\beta$$

where

$$C = \frac{1}{\sum_k w_k}\sum_k w_k\cos\beta_k$$

$$S = \frac{1}{\sum_k w_k}\sum_k w_k\sin\beta_k$$

$$r = \sqrt{C^2 + S^2}$$

- mean values of $(\alpha_G, \delta_G, V_G, \lambda_\oplus)_m$ are used to calculate $(a, q, e, \omega, \Omega, i)_m$

Our proposition

- Using least squares method with two contrains we calculated mean vectorial parameters, i. e. $\vec{G}$, $\vec{e}$ and h.
- we must remember that between $\vec{G}$ and $\vec{e}$ must be fulfiled orthogonal conditions:

$$\vec{G} \cdot \vec{e} = 0 \qquad (1)$$

- and

$$e^2 = 1 + \frac{2h}{\mu^2} G^2 \qquad (2)$$

Our proposition

Let $O_k = (G_{k1}, G_{k2}, G_{k3}, e_{k1}, e_{k2}, e_{k3}, h_k)^T$ be set of N meteor orbit.

Then, $O_s = (G_{s1}, G_{s2}, G_{s3}, e_{s1}, e_{s2}, e_{s3}, h_s)^T$ is the mean orbit of the stream, and for which are fulfiled conditions (contrains) from previous slide.

Vectorial elements of mean orbit we find by least squares method applied to linear form of conditions and constrains equations.

$$\mathbf{v} - \mathbf{I}\mathbf{O_s} = -\mathbf{O},$$

$$\mathbf{v} - \mathbf{I}\boldsymbol{\Delta}\mathbf{O_s} = -\mathbf{O} + \mathbf{I}\mathbf{O_{so}},$$

$$\mathbf{C}\boldsymbol{\Delta}\mathbf{O_s} = \mathbf{g_o}$$

Our proposition

Those equations have to be solved with condition

$$\Phi = \mathbf{v}^T \mathbf{W} \mathbf{v} - 2\mathbf{L}_c^T (\mathbf{C} \Delta O_s - \mathbf{g}_0) \rightarrow min$$

where **W** is weights matrix of vectoral elements, and $\mathbf{L}_c$ is vector of Lagrange factors.
Solution of above equation is given by

$$\Delta \mathbf{O}_s = \mathbf{R}^{-1} \mathbf{t}$$

Having approximation of $O_{s0} = (\mathbf{G}_o, \mathbf{e}_o, h_o)^T$ and corrections $\Delta \mathbf{O}_s$

we can find O_s by iteration procedure.

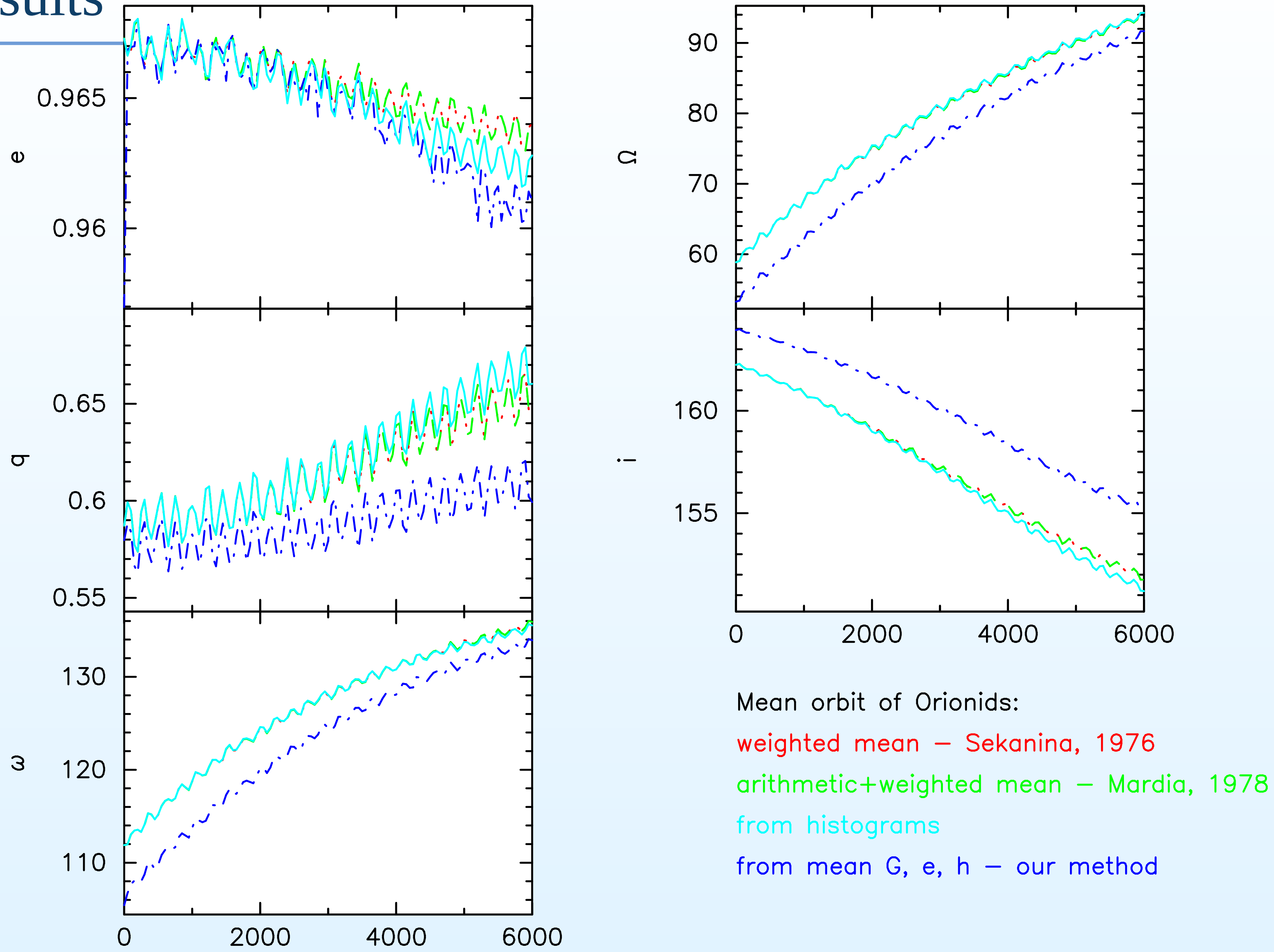
Results

Stream name	M	Dates		a (AU)	q (AU)	e	i (deg)	ω (deg)	Ω (deg)
Lyrids	11	21 Apr	25 Apr	27.150	0.920	0.966	79.6	213.9	32.3
				27.631	0.921	0.967	79.6	213.9	32.3
α Capricornids (N)	22	20 Jul	10 Aug	2.520	0.586	0.768	7.4	268.3	128.0
				2.548	0.586	0.770	7.4	268.4	127.9
Perseids	307	30 Jul	22 Aug	26.602	0.953	0.964	113.1	152.1	139.0
				28.966	0.955	0.967	113.2	152.1	139.0
κ Cygnids	23	4 Aug	22 Aug	3.791	0.973	0.743	35.1	202.9	141.2
				3.869	0.975	0.748	35.1	202.9	141.1
Taurids (N)	33	16 Oct	24 Nov	1.973	0.339	0.828	3.4	295.5	223.6
				2.123	0.339	0.841	3.4	296.3	222.7
Taurids (S)	50	19 Oct	22 Nov	2.104	0.365	0.827	5.3	113.0	41.4
				2.222	0.367	0.836	5.3	112.7	41.5
Quadrantids	52	2 Jan	4 Jan	3.164	0.978	0.691	72.0	171.1	283.3
				3.173	0.978	0.692	72.1	171.1	283.3

Results

Stream name	M	Dates		a (AU)	q (AU)	e	i (deg)	ω (deg)	Ω (deg)
Geminids	279	7 Dec	16 Dec	1.357	0.140	0.897	23.9	324.4	262.2
				1.365	0.140	0.897	23.9	324.4	262.2
Leonids	33	16 Nov	20 Nov	11.239	0.983	0.913	161.6	172.1	235.2
				11.426	0.983	0.914	161.7	172.3	235.3
Orionids	32	18 Oct	28 Oct	17.183	0.575	0.967	164.1	81.7	28.8
				17.765	0.575	0.968	164.1	81.9	28.9
S. δ Aquarids	16	22 Jul	9 Aug	2.663	0.082	0.969	25.9	149.9	309.7
				2.782	0.084	0.970	26.4	149.9	309.6

Results


e
0.965
0.96
q
0.65
0.6
0.55
ω
130
120
110
0
2000
4000
6000
Ω
90
80
70
60
i
160
155
0
2000
4000
6000
Mean orbit of Orionids:
weighted mean – Sekanina, 1976
arithmetic+weighted mean – Mardia, 1978
from histograms
from mean G, e, h – our method